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ON THE CONSTRUCTION OF GROUP DIVISIBLE FAMILY OF DESIGNS

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ABSTRACT

Group divisible designs, rectangular designs and nested group divisible designs which form a group divisible family of designs is very important class of designs. Construction of group divisible family of designs will be surveyed here.

1. Preliminaries

1.1. Group divisible design

A group divisible (GD) design is a 2-associate PBIB design based on a group divisible association scheme, *i.e.*, a set of the mn treatments can be divided into m groups of n treatments each such that any two treatments occur together in λ_1 blocks if they belong to the same group, and in λ_2 blocks if they belong to different groups.

1.2. Rectangular design.

A rectangular design is a 3-associate PBIB design based on a rectangular association scheme of v = mn treatments arranged in an m x n rectangle such that, with respect to each treatment, the first associates are the n-1 (= n_1, say) treatments of the same row, the second associates are the other m-1 (= n_2 ,say) treatments of the same column and the remaining (m-1)(n-1) (= n_3, say) treatments are the third associates.

That is a rectangular design in arrangement of v = mn treatments in b blocks such that (i) each block contains k distinct treatments, k <v (ii) each treatment occurs in exactly r blocks (iii) the mn treatments are arranged in a rectangle of m rows and n columns such that any two treatments in the same row(column) occur together in λ_1 (λ_2) blocks, respectively, and in λ_3 blocks, otherwise. When $\lambda_2 = \lambda_3$ a rectangular design reduces to a group divisible design.

1.3. Nested group divisible design

A nested group divisible design is an arrangement of v = mnp treatments divided into p sets of mn treatments each and b blocks of size k, satisfying the following:

(I) Each of p sets consists of m groups of size n; and any two treatments (i) in the same group and same set are called first associates(ii) in different groups and same set are called second associates (iii) otherwise, called third associates, (II)Each treatment is repeated r times,

(III) Any two treatments which are i-th associates occur together in λ_i blocks for i =1,2, 3.

When p=2 and $\lambda_2 = \lambda_3$ a nested group divisible design also reduces to a group divisible design.

Thus rectangular designs and nested group divisible designs are both reducible to group divisible designs as special cases. Therefore these designs may be called group divisible family of designs.

2. Description

BIB designs were comprehensively dealt with in Bose (1939).Although BIB designs are universally optimal with respect to wide range of criteria, they are available only for a limited parameteric values of (v, b, r, k) and that to require high values of r. To meet this situation Bose and Nair (1939) introduced the concept of partially balanced designs by relaxing the condition of constancy of λ.

Further Bose and Shimamoto (1952) classified these designs. Later on, Bose, Clatworthy, and Shrikhande (1954) tabulated two associate class partially balanced incomplete block designs in the range of r, k<10.

Further Clatworthy (1973) comprehensively revised these tables which includes group divisible, triangular, latin square and cyclic designs. Various methods of construction of these designs are given in Raghavarao (1971).

2.1. Group-divisible designs

New GD designs

Clatworthy's tables contain 443 parameteric combinations of group divisible (GD) designs with their solutions. Since then Freeman (1976), Dey

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(1977), John and Turner (1977), Bhagwandas and Parihar (1980 ,1982), Dey and Nigam (1985), Sinha (1987,1989), De and Roy(1990) reported many new group divisible designs not found in the tables of Clatworthy (1973). Sinha (1991) listed the new group divisible designs reported after 1973.

Any body reporting a new GD design in the range of r, k<10 will be credited with online reference in : (https://sites.google.com/site/kishoresinha).

Methods of constructions

Apart from the methods of constructions reported by authors referred above, Several other methods of constructions of group divisible designs were given by Kageyama and Tanaka (1981), Kageyama (1985a,1985b) Kageyama and Mohan (1985a,1985b), Banerjee and Kageyama (1986) , Sinha and Kageyama (1986, 1989), Sinha (1991, 1994), Mitra,Sinha,Saha,Singh (2002), Mitra, Sinha, Mandal, Kageyama (2002), Sinha and Kageyama (2006).

It is well known that semi-regular group divisible designs are related to orthogonal arrays.

It was shown in Sinha and Nigam (1982) that regular group divisible designs are related to balanced arrays.

2.2. Rectangular designs.

Rectangular designs introduced by Vartak (1955) form a special class of 3-associate Partially

balanced incomplete block (PBIB) designs based on a rectangular association scheme. The rectangular association scheme is also called 3-associate Kronecker Product association

Scheme or a 3-associate extended group divisible (EGD) association scheme. These designs have been earlier studied among others by Hinkelmann (1964), Raghavarao and Aggarwal (1974), Kageyama and Tanaka (1981), Suen (1989).

Several series of rectangular design were constructed in Sinha, Kageyama, Singh (1993), Mitra, Sinha, Saha (1996), Sinha and Mitra (1999), Mitra,Sinha,Mandal,Kageyama (2002),

Sinha ,Singh,Kageyama,Singh(2002).

The rectangular designs are useful as factorial experiments having balance as well as orthogonality.

Prasad (2007) established equivalence between extended group divisible (EGD) designs and designs for crop sequence experiments and some real life applications of EGD designs were also given.

Rectangular designs with varying replicates were considered in Das,Sinha,Kageyama(1992).

Rectangular designs are also related to balanced arrays,seeSinha,Dhar,Saha,Kageyama(2002)

2.3. Nested group divisible designs

Several methods of construction of nested group divisible designs have been described by Sinha, Bhagwandas, Kageyama (1992), Duan and Kageyama (1993), Miao, Kageyama, Duan(1996), Sinha (1999), Mitra, Sinha, Kageyama, Singh (2002).

Sinha and Shah (1988) established the E-optimality of a class of designs called 3-concurrence most balanced designs. A nested group divisible design which are a special class of 3-concurrence most balanced designs, having v=2mp, and λ_1, $\lambda_2=\lambda_1+2$, $\lambda_3=\lambda_1+1$ is E-optimal.

Sinha and Kageyama (1992), Sinha (1994) constructed some series of E-optimal nested group divisible designs.

The nested group divisible designs are also useful as factorial experiments having balance as well as orthogonality.

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