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# REDUNDANCY ALLOCATION IN A SYSTEM: A BRIEF REVIEW

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**ABSTRACT**

*Redundancy allocation is one of the widely used techniques to improve the reliability of a system. Here, we discuss different types of standby (e.g. cold, hot and warm) allocations in different types of systems, including some recent developments in this field.*

***Keywords***: Cold standby, hot standby, reliability, stochastic orders, warm standby.

## 1. Introduction

It is a fundamental doctrine in reliability analysis that if no compensatory measures are undertaken, the reliability of a system decreases with increase in complexity of the system. Mathematically, the reliability is defined as the probability that a system will perform its intended function satisfactorily, for a specified period of time, under a given set of conditions. Note that, in a (series) system of $n$ independent components with ith component having reliability $R_i$, if no component is redundant, then the system reliability ($R$) will be $R = \prod_{i=1}^{n} Ri$.

The redundant (also known as standby or spare) components could be of three types: (i) Hot (also known as active or parallel) standby (ii) Cold standby and (iii) Warm standby. In hot standby, the original component and the redundant component work together in the same operational environment so that the life of the system is the maximumof the lives of the original component and the redundant component. Thus, the original component and the standby component constitute a two-component parallel system. In cold standby, the redundant component starts working once the original component fails. Here the redundant component has zero failure rate when it is in inactive state and it starts to function under the usual environment (in which the system is running) only when the original component fails. Thus, the system life is the convolution of the lives of the original component and the redundant component. It is generally assumed here that the standby component does not fail in the cold state. However, in some practical situations this may not be the case and the modelling may be done accordingly. For example, a battery put on cold standby deteriorates with time if it is not used for long, and as a result, it may fail even before the original battery fails. On the other hand, an intermediate scenario is described in warm standby which is also known as general standby because it gives hot standby and cold standby as particular cases. Here the redundant component undergoes two operational environments: initially it functions in a milder environment where the standby has less failure rate than its actual failure rate, and then it switches over to the usual environment after the original component fails. If it is known a priori (from some practical experience or otherwise) that the active (original) component cannot fail before some specified time t0, then a warm standby may undergo three operational environments. To be more specific, initially it may start as cold standby (*i.e.*, it has zero failure rate during the period $[0, t_0]$), it is switched over to warm state after $t_0$, and put into active state in usual environment at the time of failure of the original component.

Warm standby is used where the switching time from the failed component to the standby component cannot be allowed. In this case, the standby component is kept in a low-charging state so that the failure rate is neither zero (as in case of cold standby) nor same as the failure rate of the original component (as in case of hot standby), but a state where failure rate is positive but small so that once the component fails, the standby component starts working immediately. This is used in case of shadowless lamp used in case of surgery, *etc*. In this case, the 'censoring and switching device' is not allowed to take any time to start operating. It is obvious from the above discussion that, in general, the cold standby system will have the largest lifetime followed by the lifetime of a warm standby system, and the hot standby system will have the smallest lifetime. So, one may argue that we can always use cold standby to optimize the system lifetime. This may not be so in all practical cases. In some situations, it is quite possible that the redundant component needs warm up time before it starts working. (In some play, the players may wait outside the field and warm up themselves until they are asked to

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play in place of some player). Thus, depending on the practical need, one may decide on what kind of standby to be used.

Let us consider a single-component system having lifetime $X_0$ with $k$ cold standby components (with the $i$th standby having the lifetime $X_i$, $i = 1, 2, \ldots, k$). Then the system reliability at some specified time $t$, $R(t)$, is given by

$$R(t) = P(X_0 + X_1 + \ldots + X_k \geq t).$$

If Xi has distribution function $F_i$, $i = 0, 1, \ldots, k$, then the calculation of $R(t)$ becomes cumbersome unless Fis are identical for all i. In case they are not identical, we may sometimes use Laplace transform technique to evaluate $R(t)$. If the calculation becomes intractable analytically, we may go for Monte Carlo simulation technique.

In the above example, if the original component and the k redundant components are put in a hot standby system, the calculation becomes easier. This is because the reliability of such a hot standby system is

$$R(t) = P(\max\{X_0, X_1, \ldots, X_k) \geq t)$$

$$= 1 - \prod_{i=0}^{k} F_i(t).$$

For discussions on hot standby and on cold standby systems, one may refer to Boland et al. (1992), Singh and Misra (1994), Romera *et al.* (2004), Papageorgiou and Kokolakis (2007), Brito *et al.* (2011), Misra *et al.* (2011), and the references therein.

As far as analytical development is concerned, the warm standby system, being more general in the sense that the cold standby and the hot standby systems are obtained as particular cases of the warm standby system, is complicated. Although She and Pecht (1992) have done some work on warm standby model, to the best of our knowledge, Cha *et al.* (2008) are the first to work on warm standby models incorporating the accelerated life model (cf. Nelson, 1990) and the virtual life model (cf. Kijima, 1989; Finkelstein, 2007) in their analysis, followed by the work of Yun and Cha (2010), Li *et al.* (2009, 2013), Eryilmaz (2013) and others.

Let $X$ be the lifetime of the original component with cumulative distribution function (c.d.f.) $F(\cdot)$ and let $Y$ be that of the spare with c.d.f. $G(\cdot)$, with $X$ and $Y$ independent. Consider a system where $Y$ is allocated to $X$ to form a warm standby system. The life of such a system is denoted by $X \circledast Y$. For a standby component in warm state, it is obvious that the lifetime of the standby component in the milder environment is stochastically larger than that in the usual environment. Thus, based on the concept of accelerated life model, the lifetime of the standby unit in warm state will have the c.d.f. G($\gamma(\cdot)$) where $\gamma(\cdot)$ is a non-decreasing function satisfying $\gamma$(t) $\leq$ t, for all t $\geq$ 0 with $\gamma$(0) = (0). Further, suppose that the standby unit has worked during (0, t) without failure in the milder environment, and is activated under usual environment at time t. Then, according to the virtual age model, the virtual age $\omega$(t) of the standby component should be non-decreasing satisfying $\omega$(t) $\leq$ t, for all t $\geq$ 0 and $\omega$(0) = 0. Let $Y^*$ denote the remaining lifetime of the standby component after the failure of the active unit at time $X$=$x$. Now, if it is known a priori that the standby component survives at least up to the failure time of the active component, then (cf. Cha *et al.* (2008)).

$$P\{Y^* > t \mid X = x\} = \frac{\bar{G}(\omega(x)+t)}{\bar{G}(\omega(x))}\bar{G}(\gamma(x))$$

and the reliability of the standby system is

$$\bar{F}_{x \circledast y}(t) = \bar{F}(t) + \int_0^t \frac{\bar{G}(\omega(x)+t-x)}{\bar{G}(\omega(x))}\bar{G}(\gamma(x))dF(x). \tag{1.1}$$

The cold and the hot standby models can be derived as special cases by substituting $\gamma(t) = \omega(t) = 0$ and $\gamma(t) = \omega(t) = t$, respectively.

Now, as discussed above, if the standby component starts to work as cold standby, is switched over to warm state after pre-specified time $t_0$, and is put into active state in usual environment at the time ($X = x$) of failure of the original component, then obviously the virtual age at time $x$ would be $\omega(x - t_0)$. Then, for $t > t_0$, the reliability of the system is (cf. Yun and Cha, 2010),

$$\bar{F}_{x \circledast y}(t) = \bar{F}(t) + \int_{t_0}^t \frac{\bar{G}(\omega - t_0 +) = t - x)}{\bar{G}(\omega(x - t_0))}\bar{G}(\gamma(x - t_0))dF(x). \quad (1.2)$$

In the following, we discuss reliability properties of various multi-component system equipped with standby components. Here we give some definitions of stochastic orders that will be used in sequel. For details of these orders, one may refer to Shaked and Shanthikumar (2007).

**Definition 1.1** *Let X and Y be two absolutely continuous random variables with c.d.f.* $\bar{F}(\cdot)$, $\bar{G}(\cdot)$, *probability density functions f(·), g(·), hazard rate functions* $\lambda_1(\cdot)$, $\lambda_2(\cdot)$, *and the corresponding reversed hazard* *rate functions $r_1(\cdot)$ and $r_2(\cdot)$, respectively. Then X is said to be smaller than Y in the*

*(i) usual stochastic order (denoted as $X \leq_{st} Y$) if $\bar{F}(t) \leq \bar{G}(t)$ for all t;*

*(ii) failure (hazard) rate order (denoted as $X \leq_{hr} Y$) if $\bar{G}(t)/\bar{F}(t)$ is increasing in te $\geq 0$, or equivalently, if $\lambda_1(t) \geq \lambda_2 2(t)$ for all t $\geq 0$;*

*(iii) reversed hazard rate order (denoted as $X \leq_{rh} Y$) if G(t)/F (t) is increasing in t > 0, or equivalently, if $r_1(t) \leq r_2(t)$ for all t > 0;*

*(iv) increasing concave order (denoted as $X \leq_{icv} Y$) if $\int_0^t \bar{F}(x)dx \leq \int_0^t \bar{G}(x)dx$ for all $t \geq 0$;*

*(v) likelihood ratio order (denoted as $X \leq_{lr} Y$) if f(x)/g(x) decreases in x over the union of the supports of X and Y ;*

*(vi) probability order (also known as stochastic precedence order) (denoted as $X \leq_{pr} Y$) if P (X > Y) $\leq$ P (Y > X).*

Throughout the paper we use $X =_{st} Y$ to mean that the random variables *X* and *Y* have the same distribution.

## 2 System equipped with standby components

Throughout the paper, *T* denotes the lifetime of a coherent system without standby and $X_i$ denotes the lifetime of the *i*th component in the system, for *i* = 1, 2, ..., n. Let *F* be the common absolutely continuous c.d.f. of $X_i$ with corresponding p.d.f. *f* . It is known that the survival function of a coherent system formed from the independent and identically distributed (*iid*) components can be represented by the system signature as

$$P\{T > t\} = \sum_{i=1}^{n} p_i \, P\{X_{i:n} > t\},$$

where $X_{i:n}$ is the *i*th smallest order statistic from $X_1, X_2, ..., X_n$, and $p_i = P\{T = X_{i:n}\}$ is the probability that the *i*th component failure causes the system to fail. The *n*-dimensional probability vector $p = (p_1, p_2, ..., p_n)$ is called the signature of the system (cf. Kocher *et al.*, 1999; Samaniego, 1985, 2007).

### 2.1. System equipped with cold standby component

Boland *et al*. (1992) investigated the problem of where to allocate a standby in a system in order to stochastically optimize the lifetime of the resulting system. They showed that if the life distributions of the components are ordered in the likelihood ratio order, then in a series (resp. parallel) system the standby (cold) allocation should go to the weakest (resp. strongest) component to stochastically maximize the system lifetime. That is, if $X_1, X_2$ (independent and non-negative random variables) are the lifetimes of two components and *Y* (independent of $X_1$ and $X_2$) is the lifetime of a spare component, then

$$T_1 = \min\{X_1 + Y, X_2\} \geq_{st} \min\{X_1, X_2 + Y\} = T_2$$

provided $X_1 \leq_{hr} X_2$, and

$$U_1 = \max\{X_1 + Y, X_2\} \leq_{st} \max\{X_1, X_2 + Y\} = U_2$$

provided $X_1 \leq_{rh} X_2$. They have also pointed out that $T_1 \geq_{st} T_2$ if, and only if, the densities or mass functions of $X_1$ and $X_2$ belong to a one-parameter family with monotone likelihood ratio property. However, they showed with an example that the condition on the hazard rate and the reversed hazard rate orders cannot be replaced by the condition on the usual stochastic order to yield the same results. With an example of a 2-out-of-3 system, they also illustrated that similar results cannot be extended to the more general k-out-of-n system. Singh and Misra (1994) compared the lifetimes of series and parallel systems with a standby redundancy in probability order. Let $X_1, X_2, ..., X_n$ be the independent lifetimes of n components and let

$$T_1^n = \min\{X_1 + Y, X_2, ..., X_n\},$$

$$T_2^n = \min\{X_1, X_2 + Y, X_3, ..., X_n\},$$

$$U_1^n = \max\{X_1 + Y, X_2, ..., X_n\}$$

$$U_2^n = \max\{X_1, X_2 + Y, X_3, ..., X_n\}.$$

Singh and Misra (1994) established that if $X_1 \leq_{st} X_2$, then $T_2^n \leq_{pr} T_1^n$ and $U_1^n \leq_{pr} U_2^n$. El-Neweihi and Sethuraman (1993) considered a system where, instead of one spare, there are *n* spares to be assigned, one each, in standby redundancy to the *n* components, and obtained optimal allocation results for both series and parallel systems. They established that, for a series system, if the components are ordered in the hazard rate ordering and the spares are also ordered in the hazard rate ordering, then it is optimal to allocate the spares to the components in reverse order, that is, the stronger spares should be allocated to the weaker components in order. In case of parallel system, if both the components and the spares are ordered in reversed hazard rate ordering, then the spares should be allocated to the components in order, that is the stronger spares should be allocated to the stronger components in order.

Eryilmaz (2012) presented some stochastic ordering results for the lifetime of a *k*-out- of-*n* system with a single cold standby component. In a *k*-out-of-*n* system with a single cold standby unit, at the time of system failure, i.e., at the time of ($n$ " $k$ + 1)th failure, the standby component is put into operation. Thus, the lifetime of a *k*-out-of-*n* system with single cold standby unit having lifetime Z can be represented as

$$T = \begin{cases} X_{n\text{-}k+1:n} + \min(X_{n\text{-}k2:n} - X_{n\text{-}k+1:n}, Z), & \text{for } k=2, ..., n \\ X_{n:n} = Z & \text{for } k=1, ..., n \end{cases}$$

Define

$$T_i = X_{n\text{-}k+1:n} + \min(X_{n\text{-}k+2:n} - X_{n\text{-}k+1:n}, Z_i) \; i = 1, 2$$

where $Z_1$, $Z_2$ are the lifetimes of two different cold standby components. He also derived that if $Z_1 \leq_{st} Z_2$, then

(i) $T_1 \leq_{st} T_2$;

(ii) $E(T_1 - t \mid X_{n\text{-}k+1:n} > t) \leq E(T_2 - t \mid X_{n\text{-}k+1:n} > t)$.

If the lifetimes of the components are exponentially distributed and $Z_1 \leq_{hr} Z_2$, then $T_1 \leq_{hr} T_2$.

Papageorgiou and Kokolakis (2007) evaluated reliability of a two-unit parallel system with (n – 2) cold standby components where two units start their operation simultaneously and any one of them is replaced instantaneously upon its failure by one of the (n – 2) cold standbys. Coit (2001) obtained reliability of a series-parallel system with cold standby redundancy considering both perfect and imperfect switching cases, and developed a methodology to determine optimal design configurations to maximize the system reliability. Here perfect switching means perfectness of a detection and switching mechanism used to detect failure of a component and to activate a redundant component, if any, and imperfect switching means that there is a probability of failure of the detection and switching mechanism. van Gemund and Reijns (2012) studied the *k*-out-of-*n* system with a single cold standby component using Pearson-type distributions and presented an analytical approach to compute the mean failure time of the system. Reliability function of a *k*-out-of-*n* system equipped with a cold standby component is given by (cf. Eryilmaz, 2012)

$$P\{T^{cs} > t\} = P\{X_{n\text{-}k+1:n} > t\} + \frac{\bar{F}^{k-1}(t)}{B(n-k+1,k)} \int_0^t \bar{G}(t-x) F^{n-k}(x) dF(x),$$

where $T^{cs}$ denotes the lifetime of the *k*-out-of-*n* system with cold standby having continuous c.d.f. G( ·). He derived three different mean residual life functions under three conditions, for a *k*-out-of-*n* system with a single cold standby component using dis- tributions of order statistics. Eryilmaz (2014) derived reliability of a coherent system equipped with a cold standby component which may be put into operation at the time of the first component failure in the system. So, the results obtained in that paper are useful for a coherent system if the system has a probability of failure at the time of the first component failure. Later, Franko *et al.* (2015) generalized this case by considering that standby component may be put into operation at the time of the sth component failure, s = $k_{\varnothing}, k_{\varnothing}+1, ..., z_{\varnothing+1}$, where $k_{\varnothing}$ is the minimum number of failed components that causes the system failure, whereas $z_{\varnothing}$ is the maximum number of failed components so that system can still operate. Obviously, in this case, the system has a probability of failure at the time of the sth component failure.

## 2.2 System equipped with hot standby component

Boland *et al.* (1988) introduced a measure of component importance (termed as redundancy importance) for a coherent system in which one or more active redundant component(s) is/are allocated at the component level. The measure 'redundancy importance' of a component is defined in that paper as the improvement in reliability of the system by allocating an active redundancy with that component. They compared redun- dancy importance of a component with (Birnbaum) reliability importance and structural importance of a component in a coherent system. Boland *et al.* (1992) considered a two-component series system and formed two systems — one by allocating an active redundancy in parallel with the first component, and another by allocating the same active redundancy with the second component. They discussed the conditions under which one system is stochastically more favorable than another as follows. Let us consider a series system consisting of two components having lifetimes $X_1$ and $X_2$ which are independent and non-negative. Also assume that a spare component having lifetime $Y$ (independent of $X_1$ and $X_2$) is available for active redundancy with one of the components in the system. Let

$$^* = \min\{\max(X_1, Y), X_2\}$$

and

$$^* = \min\{X_1, \max(X_2, Y)\}.$$

Boland *et al.* (1992) proved that $X_1 \leq_{st} X_2$ if, and only if, $T_1^* \geq_{st} T_2^*$. They also considered the problem of allocating a redundant component in a $k$-out-of-$n$ system with independent components. Let $X_1, X_2, ..., X_n$ be the lifetimes of n independent components and let

$$X_{[k]} = \{X_1, X_2, ..., X_n\}_{[k]}$$

denote the $k$th largest order statistic so that $X_{[1]} \geq X_{[2]} \geq ... \geq X_{[n]}$. If $T_{k:n}$ denotes the lifetime of a $k$-out-of-$n$ system, then $T_{k:n} = X_{[k]} = \{X_1, X_2, ..., X_n\}_{[k]}$. Then, for an active redundancy with lifetime $Y$, independent of $\{X_1, X_2, ..., X_n\}$, Boland *et al.* (1992) showed that $X_1 \leq_{st} X_2$ if, and only if,

$$\{\max(X_1, Y), X_2, ..., X_n\}_{[k]} \leq_{st} \{X_1, \max(X_2, Y), ..., X_n\}_{[k]},$$

for $k = 1, 2, ..., n$. So, this result concludes that, it is stochastically optimal always to allocate the active redundant component to the stochastically weakest component for stochastically ordered component lifetimes. Singh and Misra (1994) considered the same two-component series system with active redundancy as Boland *et al.* (1992), and showed with an example that unlike stochastic ordering, in general, the lifetime of the two systems cannot be compared in failure rate ordering. They extended the above mentioned results of Boland *et al.* (1992) to stochastic precedence order. They established that $X_1 \leq_{st} X_2$ is a sufficient condition for $T^* \geq_{pr} T_2^*$ to hold. They also showed that if $X_1 \leq_{st} X_2$ then for $k = 1, 2, ..., n$,

$$P(\{\max(X_1, Y), X_2, ..., X_n\}_{[k]} > \{X_1, \max(X_2, Y), ..., X_n\}_{[k]})$$
$$\geq P(\{X_1, \max(X_2, Y), ..., X_n\}_{[k]} > \{\max(X_1, Y), X_2, ..., X_n\}_{[k]}).$$

Valdés and Zequeirab (2003) considered the same systems as of Boland *et al.* (1992) but with two different active standby components, and compared the lifetimes of the two systems in stochastic order and in failure rate order. With $Y_1$ and $Y_2$ as lifetimes of two different spares, independent of components having lifetimes $X_1$ and $X_2$, they compared the lifetimes of $U_1^*$ and $U_2^*$ defined as

$$U_1^* = \min\{\max(X_1, Y_1), X_2\} \text{ and}$$

$$U_2^* = \min\{X_1, \max(X_2, Y_2)\}.$$

They showed that if either of the sets of conditions

(a) $\{X_1 \leq_{st} X_2, Y_1 \geq_{st} Y_2\}$

(b) $\{X_1 \leq_{st} X_2, X_1 \leq_{st} Y_1, X_2 \geq_{st} Y_2\}$

holds, then $U_1^* \geq_{st} U_2^*$. Let us denote the failure rate functions of the random variables $X_1, X_2, Y_1$ and $Y_2$, respectively, by $\lambda_1(\cdot)$, $\lambda_2(\cdot)$, $\mu_1(\cdot)$ and $\mu_2(\cdot)$. They showed that if

(i) $\lambda_2(t) > 0$ for $t > 0$,

(ii) $\lambda_1(t) \geq \max\{\lambda_2(t), \mu_1(t)\}$,

(iii) $\mu_2(t) \geq \mu_1(t)$, for $t \geq 0$,

(iv) $\alpha(t) = \lambda_2(t)/\lambda_1(t)$ is non-increasing in $t>0$,

then $U_1^* \geq_{hr} U_2^*$. Romera *et al.* (2004) also considered the same systems with two different active redundant components, and found sufficient conditions under which one system dominates another in probability order. Suppose $\bar{F}_1(\cdot)$, $\bar{F}_2(\cdot)$, $\bar{G}_1(\cdot)$ and $\bar{G}_2(\cdot)$ are the survival functions of $X_1, X_2, Y_1$ and $Y_2$ respectively. They showed that if

(i) $X_1 \leq_{st} X_2$

(ii) $\bar{F}_2(x)\bar{G}_1(x) \geq \bar{F}_1(x)\bar{G}_2(x)$, $x \geq 0$,

then $U_1^* \geq_{pr} U_2^*$. They also formed two systems by allocating both the spares with each component alternatively so that the lifetimes become

$$V_1 = \min\{\max(X_1, Y_1), \max(X_2, Y_2)\}$$

and

$$V_2 = \min\{\max(X_1, Y_2), \max(X_2, Y_1)\},$$

and showed that

$$X_1 \leq_{hr} X_2, Y_1 \leq_{hr} Y_2 \Longrightarrow V_1 \geq_{pr} V_2.$$

They also considered the allocation of active redundancy to $k$-out-of-$n$ system and obtained similar results. Valdés and Zequeira (2006) compared the lifetimes $V_1$ and $V_2$ in the failure rate order. If

(i) $(t) > 0$, $t > 0$,

(ii) $X_1 \leq_{hr} X_2$,

(iii) $\alpha(t)$ is non-increasing,

then $V_1 \leq_{hr} V_2$. They also showed that

$$\lambda_1(t) \geq 2\lambda_2(t) \Longrightarrow V_1 \leq_{hr} V_2.$$

Let $r_1(\cdot)$ and $r_2(\cdot)$ denote the reversed hazard rate functions of $X_1$ and $X_2$ respectively. Brito *et al.* (2011) showed that if

(i) $X_i$=st $Y_i$, $i = 1, 2$,

(ii) $X_1 \leq_{rh} X_2$,

(ii) $\gamma(t) = r_1(t)/r_2(t)$ $(t > 0)$ is increasing,

then $V_1 \leq_{rh} V_2$. It is observed that if $\gamma(t) \leq 1/2$, then the above result is satisfied without any condition on $\gamma$ (t). Further, if $X_i =_{st} Y_i$, $i$ = 1, 2, and $r_1$, $r_2$ are proportional, then $V_1 \leq_{rh} V_2$ if and only if $X_1 \leq_{rh} X_2$.

Valdės *et al*. (2010) considered an n-component series system and formed two systems by allocating an active redundancy in parallel with the first component, and by allocating another active redundancy with the second component. They compared the lifetimes of such series systems in increasing concave order. Let $X_1$, $X_2$, ..., $X_n$ be the lifetimes of n independent components, and let $Y_1$, $Y_2$ be those of the two independent spares. Write

$$U_1 = \min\{\max(X_1, Y_1), X_2, Z\}$$

and

$$U_2 = \min\{X_1, \max(X_2, Y_2), Z\},$$

where $Z = \min\{X_3, X_4, ..., X_n\}$. Then, Valdės *et al*. (2010) derived that if any one of

(a) $X_1 \leq_{icv} X_2$, $Y_1 \geq_{st} Y_2$

(b) $X_1 \leq_{icv} X_2$, $X_1 \leq_{st} Y_1$, $X_2 \geq_{st} Y_2$

holds, then $U_1 \geq_{icv} U_2$. They also compared the lifetimes of series systems formed by allocating two different active redundancies at the first two components alternatively, *i.e*., by writing

$$V_1 = \min\{\max(X_1, Y_1), \max(X_2, Y_2), Z\}$$

and

$$V_2 = \min\{\max(X_1, Y_2), \max(X_2, Y_1), Z\}.$$

They showed that if any one of

(a) $X_1 \leq_{hr} X_2$ and $Y_1 \geq_{hr} Y_2$,

(b) $X_1 \leq_{hr} X_2$ and $Y_1 \geq_{hr} Y_2$

holds, then $V_1 \geq pr\ V_2$. Now, suppose $X_i = st\ Y_i$, $i = 1, 2$. If any one of

(a) $X_1 \leq_{hr} X_2 (X_1 \leq_{rh} X_2)$

(b) $X_2 \leq_{hr} X_1 (X_2 \leq_{rh} X_1)$,

holds, then $V_1 \leq_{pr} V_2$. Misra et al. (2011) also considered the same systems as those of Vald´es et al. (2010) and provided some additional conditions under which one system dominates the other in stochastic precedence order. They obtained that if one of the conditions

(a) $X_1 \leq_{st} X_2$ and $Y_2 \leq_{st} X_1(X_2) \leq_{st} Y_1$,

(b) $X_1 \leq_{st} X_2$, $X_1 \leq_{st} Y_1$, and $Y_2 \leq_{hr} Y_1$,

(c) $X_1 \leq_{st} X_2$, $Y_2 \leq_{st} X_2$, and $Y_2 \leq_{rh} Y_1$,

(d) $X_2 \leq_{st} X_1$ and $Y_1 \leq_{st} X_1(X_2) \leq_{st} Y_2$,

(e) $X_2 \leq_{st} X_1$, $X_2 \leq_{st} Y_2$, and $Y_1 \leq_{hr} Y_2$

(f) $X_2 \leq_{st} X_1$, $Y_1 \leq_{st} X_1$, and $Y_1 \leq_{rh} Y_2$

holds, then $V_1 \geq_{pr} V_2$. They also obtained the conditions under which the two systems dominate one another in failure rate and in reversed failure rate orders when each spare and its associated component are stochastically equal.

### 2.3 Study of a system equipped with general standby compo- nent

After Cha *et al*. (2008) developed a general standby model for a single-component system based on the concept of accelerated life model, significant works have been done in this field. The reliability of the general standby system as obtained by Cha *et al*. (2008) is given in (1.1). Li *et al*. (2009) investigated the general standby system and derived some stochastic comparison results on the lifetimes of the systems. Denote the lifetime of a general standby system composed of an active component with lifetime $X$ and a standby component with lifetime $Y$ by $T_{X \circledast Y}$. Li *et al*. (2009) derived that

$$Y_1 \leq_{hr} Y_2 \Longrightarrow T_{X \circledast Y_1} \leq_{st} T_{X \circledast Y_2}.$$

They showed with an example that the condition on hazard rate order cannot be replaced by stochastic order to yield the same result. They also derived that if the conditions

*(i)* $(u - \omega(u))$ is increasing in $u \geq 0$,

*(ii)* $\omega'(u) \geq \gamma'(u)$ for any $u \geq 0$,

*(iii)* $Y$ is IFR (increasing failure rate)

*(iv)* $X_1 \leq_{st} X_2$

hold, then $T_{X1 \circledast Y} \leq_{st} T_{X_2 \circledast Y}$. Again, if

(i) $(u - \omega(u))$ is increasing in $u \geq 0$,

(ii) $\omega(u) = \gamma(u)$ for any $u \geq 0$,

(iii) $Y$ is DRHR (decreasing in reversed hazard rate)

(iv) $X_1 \leq_{lr} X_2$

hold, then $T_{X_1 \circledast Y} \leq_{rh} T_{X_2 \circledast Y}$. It is to be mentioned here that a random variable having survival function $\overline{F}$ and distribution function $F$ is said to be IFR (resp. DRHR) if $\overline{F}$ (resp. $F$) is log concave. Li *et al*. (2009) also compared lifetimes of two-component series system containing a general standby component. Li et al. (2013) considered a parallel system and a series system, each with two active components and one general standby, and derived some stochastic comparison results for the two systems. Suppose $X_1$, $X_2$ and $Y$ are mutually independent, and $U_1 = \max\{T_{X_1 \circledast Y}, X_2\}$ and $U_2 = \max\{X_1, T_{X21 \circledast Y}\}$.

Li *et al*. (2013) obtained that if

(a) $(u - \omega(u))$ is increasing in $u \geq 0$,

(b) $\omega(u) - \gamma(u)$ is increasing in $u \geq 0$,

(c) $Y$ is IFR

(d) $X_1 \leq_{rh} X_2$,

then $U_1 \leq_{st} U_2$. This result indicates that, for a two component parallel system, when the standby component is IFR, it is stochastically better to allocate the standby to the component with larger life (in terms of reversed hazard rate order). Now, for a series system with two components and one general standby, denote

$$W_1 = \min\{T_{x_1 \circledast y}, X_2\} \text{ and } W_2 = \min\{X_1, T_{x_2 \circledast y}\}.$$

Li *et al*. (2009) showed that

(i) $X_1 \leq_{st} X_2 \Rightarrow W_1 \leq_{pr} W_2$,

(ii) $X_1 \leq_{hr} X_2 \Rightarrow W_1 \leq_{st} W_2$.

For a series system with two components and two general standbys, write

$$Z_1 = \min\{T_{X_1 \circledast Y_1}, T_{X_2 \circledast Y_2}\} \text{and}$$

$$Z_2 = \min\{T_{X_1 \circledast Y_2}, T_{X_2 \circledast Y_1}\}.$$

Li *et al*. (2013) proved that, if

(i) $\omega(u) = \gamma(u), u \geq 0$,

(ii) $(u - \omega(u))$ is increasing in $u \geq 0$,

(iii) $X_1 \leq_{lr} X_2$,

(iv) $Y_1 \leq_{hr} Y_2$,

then $V_1 \leq_{st} V_2$. Hazra and Nanda (2014a) constructed some standby models with one and two general standby components, and compared some different series and parallel systems corresponding to the models with respect to different stochastic orders.

Papageorgiou and Kokolakis (2010) derived the reliability function of a two-component parallel system with $(n - 2)$ warm standby components, where two units start their operation simultaneously and any one of them is replaced instantaneously upon its failure by one of the $(n - 2)$ warm standbys. Yun and Cha (2010) provided a method for modeling the general standby system with a single active and a single general standby component considering perfect switching of the standby from one state to another. Here perfect switching means that switching from warm state to active state is perfect, i.e., the standby component does not fail at the time of state change, and the switch-over is instantaneous. By imperfect switching we mean that the switching from warm to active state is instantaneous but not failure-free. Eryilmaz (2013) investigated the relia- bility properties of a k-out-of-n system equipped with a single warm standby component.

Reliability of such a system is given by

$$P\{T^{gs} > t\} = P\{X_{n-k+1:n} > t\} + \frac{\bar{F}^{k-1}(t)}{B(n-k+1,k)}$$

$$\times \int_0^t \frac{\bar{G}(\omega(x)+t-x)}{\bar{G}(\omega(x))} \bar{G}(\gamma(x)) F^{n-k}(x) dF(x).$$

where $T^{gs}$ denotes the lifetime of the *k*-out-of-*n* system with warm standby. Recently Kundu *et al*. (2015) derived reliability of a coherent system equipped with a single general standby component. In their work, standby component starts to work in cold state, it is switched over to the warm state after a specified time $u$ ($\geq 0$), before which the system certainly does not fail, and it starts to work in active state in the usual environment at the time of *s*th component failure which may cause the system failure. They also considered three different switch-over cases regarding perfectness of the switching from one state to another state of the standby component.

## 3. Component redundancy versus system redundancy

It is well known that, active redundancy at the component level is superior to redundancy at the system level in stochastic orders (cf. Barlow and Proschan, 1975; Boland *et al*., 1988; Boland and El-Neweihi, 1995; Singh and Singh, 1997). Boland and El-Neweihi (1995) demonstrated with the help of counterexample that this principle does not hold in the hazard rate ordering even for a series system if the spares do not match the original components in distribution (i.e. non-matching spares). However, for a *k*-out- of-*n* system with *iid* components and spares (*i.e.* matching spares), they conclude that active redundancy at the component level is better than redundancy at the system level in the hazard rate ordering. It is to be mentioned here that a matching spare means lifetime of the spare component and that of the original component have the same distribution; otherwise the spare component is called non-matching spare.

Singh and Singh (1997) extended this result by proving that, for a *k*-out-of-*n* system with iid components and spares, active redundancy at the component level is superior to that at the system level in likelihood ratio ordering. Misra et al. (2009) proved that, for a coherent system with *iid* components and spares, component redundancy is better than system redundancy in likelihood ratio order, under a condition on the structure function of the coherent system. Hazra and Nanda (2014b) extended this result for up shifted likelihood ratio order. They proved that, for a coherent system with iid components and spares, component redundancy is superior to system redundancy in up shifted hazard rate order, under some specific conditions. Gupta and Nanda (2001) obtained a sufficient condition under which the active redundancy at the component level is superior to that at the system level in the reversed hazard rate ordering for any coherent system with iid components and spares. Hazra and Nanda (2014b) extended the result of Gupta and Nanda (2001) to up shifted reversed hazard rate order.

In case of non-matching spares, Misra *et al*. (2009) provided a condition under which the redundancy at the component level is better than the redundancy at the system level in the reversed hazard rate order, for a coherent system with iid components and iid non-matching spares. Hazra and Nanda (2014b) extended this result to the case of up shifted reversed hazard rate order, under some additional conditions. They also showed that, for a *k*-out-of-*n* system with non-*iid* components and iid spares (non-matching), redundancy at the component level is always superior to that at the system level in stochastic precedence order.

In case of standby (cold) redundancy, Shen and Xie (1991) showed that unlike active redundancy, standby redundancy at the component level is not always better than that at the system level. They showed that redundancy at the component level is better than redundancy at the system level for series systems, while the reverse is true for parallel system. Boland and El-Neweihi (1992) also obtained this kind of result for standby redundancy in stochastic order. Meng (1996) proved that a series (resp. parallel) system is the only system for which standby redundancy at the component level is always stochastically better (resp. worse) than at the system level. Hazra and Nanda (2014b) showed that for a k-out-of-n system, component redundancy is better that the system redundancy in stochastic precedence order.

## 4. CONCLUSION

In this paper, we discuss briefly the impact of different types of standby allocations in different types of systems. The present discussion includes a vast literature review on the development in this field. This paper helps the reader to overview the developments, which will help them generate new ideas to extend the existing results.

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