TRIANGULAR GENERALISED NEIGHBOUR PBIB DESIGNS IN CIRCULAR BLOCKS FOR CORRELATED OBSERVATIONS

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ABSTRACT

*In an **m class Generalised Neighbour Balanced (GNm)** design in circular blocks, any two treatments can occur together as neighbours in the design λ_i (i = 1, 2, ..., m) number of times (Misra et al.,1991; Iqbal et al., 2012; Hamad, 2014 etc.). Literature survey reveals that in existing GNm designs no association scheme is involved for any pair treatments as neighbours. In the present paper a new series of GN_2 designs from Triangular PBIB designs has been developed, which also has a same association scheme for the treatments as neighbours. The concept of merging of triangular PBIB designs and the above mentioned GN_2 designs with correlated observations will develop a new series of Triangular Generalised neighbour balanced PBIB designs with two associate classes ($TGNPBIB_2$). The definitions, properties and structural characteristics of $TGNPBIB_2$ are also developed in the paper. The blocks used in these designs are considered as circular by using border plot concept (Azais, et al. 1993; Bailey, 2003, etc.). The presence of correlation in the form of neighbour effects among the plots in agricultural experiments is used as the model given in Gill & Shukla (1985). The main advantage of $TGNPBIB_2$ designs with correlated observations is that the analysis of such designs are identical with Triangular PBIB designs. A series of $TGNPBIB_2$ designs in circular blocks has been listed. The efficiency values (A & D) of listed designs are presented for different values of ρ (-1 to +1).*

Keywords: Correlated observations, GN_2 designs, PBIB Designs, Triangular Association Scheme

1 Introduction

Rees (1967) introduced the concept of neighbour balanced design in circular blocks in serology. Since then, the concept of construction of neighbour balanced design has become an important topic in statistics and its optimality criteria been studied in a much broader way.

In a Design with Circular blocks, positions of treatments in experimental plots are considered in 2- dimensions (Left and Right). Let D (v, b, k) be a block design D with v treatments in b blocks of size k. In design D, if treatment **a** (= 1, 2,…, v) is on i^{th} plot (= 1, 2, …,k) in j^{th} block (= 1,2,…,b), then the treatment **a'** on $(i + 1)^{th}$ plot mod k is regarded as the right- neighbour of treatment **a** in that j^{th} block. On the contrarily, the treatment on $(i-1)^{th}$ plot (mod k), is considered as the left- neighbour of treatment **a** in j^{th} block . Many authors (*e.g.* Azais, *et al.* 1993; Bailey, 2003, *etc.*) used the concept of border plots at starting and at the end positions of a block. Here, the treatment in the last plot (or right end) of a block has been placed in the initial border plot and the treatment in first plot (or left end) of a block has been placed in the last border plot of the block. However, the border plot treatment effects are not considered in the analysis of the design. Thus, a circular block can also be used as a linear block by using border plots at both ends of a linear block. Therefore, each treatment in a circular block has two neighbours, one on left and another on right.

If each treatment has every other treatment as neighbour equal number of times then the design is said to be a neighbour balanced (NB) design (Iqbal *et al.* 2012). Neighbour balance (NB) is desirable if it is known or thought that the effect of a plot is influenced by its neighbouring plots, in such cases nearest neighbour analysis is considered to be more efficient than classical analysis methods (Wilkinson *et al.*, 1983; Gill and Shukla, 1985). It also provides protection against the effects of correlated observations or potentially unknown trends highly correlated with plot positions within a block (Keifer & Wynn, 1981; Cheng, 1983; Jackroux, 1998; Bailey & Druilhet, 2004; Ahmed *et al.*, 2011 and Sahu & Majumder, 2012). But the condition in the definition of NB for a design cannot be met, always. In such situations, Misra (1988) and Misra et al. (1991) proposed the concept of m class Generalised Neighbour Balanced (GNm) designs as any two treatments can occur together as neighbours in the design λ_i (i = 1, 2, …, m) number of times. When λ_i = a constant ∀ i (=1, 2,…, m) the design becomes NB. Till then, several authors (Mishra, 1988; Mishra and Chaure, 1989; Mishra *et al.* 1991; Ahmed et al., 2009; Iqbal *et al.,* 2012 *etc.*) constructed different series of GNm designs as GN2 or GN3 *etc.* All the series of the above designs are not identical in respect of neighbouring pairs.

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Actually, no particular association scheme is involved in the present literature of GNm designs. Recently, Hamad (2014) generated a series GN2 and GN3 designs (binary and non- binary) as partially neighbour balanced designs in circular blocks without any particular association scheme.

It is well established fact that apart from BIB designs, PBIB designs are the most desirable in the field of non- orthogonal block designs block designs due to its well defined association schemes. The present piece of investigation aims to construct GN2 designs with higher efficiency values having two class triangular association scheme among the treatments as neighbours using $TPBIB_2$ designs. Their advantages are the analysis and properties with correlated observations are identical (or near identical) with PBIB designs without correlated observations.

1.1 Useful definitions and model

Following are some useful definitions associated with nieghbour balanced block designs under correlated observations:

Definition 1.1: A block of treatments with border plots will be **circular** if a treatment in the left border is same as the treatment in the right end inner plot as well as if a treatment in the right border is same as the treatment in the left end inner plot. If all the blocks of a design are circular then it is a **circular design**.

Definition 1.2: A block design in circular blocks is **neighbour balanced** (NB) if every treatment occurs equally often with every other treatment as neighbours.

Definition 1.3: A block design in circular blocks will be an **m class Generalised Neighbour Balanced** (GNm) design, if any two treatments can occur together as neighbours in the design λ_i (i = 1, 2, …, m) number of times.

Definition 1.4: An m class Generalised Neighbour Balanced (GNm) design in circular blocks with any given m class association scheme will be an m class **Generalised Neighbour Partially Balanced Incomplete Block** (GNPBIBm) design with an arrangement of v treatments in b circular blocks of size k (k < v) when

1. Any treatment occurs in r circular blocks.
2. Any pair of treatments occurs together in λ_{1i} (i = 1, 2, …, m) circular blocks and same pair treatments occurs together as neighbour in λ_{2i} (i = 1, 2, …, m) circular blocks .
3. Any treatment has exactly n_i (i = 1, 2, …, m) number of i^{th} associate treatments.
4. If the treatments α and β are mutually i^{th} associates in the association scheme, then α and β occur together in λ_{1i} (i = 1, 2, …, m) blocks as well as á and â occur together as neighbour in λ_{2i} (i = 1, 2, …, m) blocks, where the integers λ_{1i} and λ_{2i} do not depend on the pair α and β and the treatments α and β are mutually i^{th} associates (i = 1, 2, …, m).

It should be noted that in a GNPBIBm design, either all λ_{1i}'s and λ_{2i}'s are unequal, or all λ_{1i}'s are unequal but λ_{2i}'s are equal or all λ_{1i}'s are equal but λ_{2i}'s are unequal. Further, the association matrices (P) with the elements p_{jk}^{i} are similar to PBIB designs.

The integers v, b, r, k, n_i, λ_{1i} and λ_{2i} are the parameters of the GNPBIBm design. Therefore, the parametric relations of a GNPBIBm design are:

1. vr = bk,
2. $\sum_i n_i = v-1$
3. $\sum_i n_i \lambda_{1i} = r(k-1)$ and
4. $\sum_i n_i \lambda_{2i} = 2r$

The proofs of the first three relations are established for any PBIB design. In a design with circular blocks any treatment in any plot of the design has two neighbours. In the design each treatment is occurring r number of times. Therefore the total number of neighbours of any treatment in the design will be 2r which is nothing but $\sum_i n_i \lambda_{2i}$.

Let us consider a class of GNPBIBm designs with *v* treatments, *b* blocks of sizes *k* ($<$ v) with correlated observations.

Let $\mathbf{Y}_{ij}$ be the response from the i^{th} plot in the j^{th} block (i = 1, 2,.., k, j = 1, 2, …, b). Moreover, the layout includes border plots at both ends of every block *i.e.*, at 0^{th} and $(k+1)^{th}$ position whose effects are not taken under analysis. A Fixed effects additive model is considered for analyzing a neighbour balanced block design having correlated observations:

$$Y = \mu 1 + X\tau + Z\beta + \varepsilon \qquad \textbf{(1.1)}$$

where, **Y** is a n × 1 vector of observations, μ is a general mean, **1** is a n × 1 vector of ones, **X** is a n × v incidence matrix of observations versus treatments, τ is a v × 1 vector of treatment effects, **Z** is a n × b incidence matrix of observations versus blocks, β is a b × 1 vector of block effects and ε is a n × 1 vector of random errors. According to Gill and Shukla (1985), ε be the error terms with mean zero and a variance & covariance matrix be **V**, such that $V^{-1} = \sigma_{\varepsilon}^{-2}\ I_b \otimes W_k$ (**I_b** is an identity matrix of order b, $\otimes$ denotes the kronecker product and **W_k** is the correlation matrix of k observations within a block). Assuming the NN_1 model of Kiefer and Wynn (1981), that there is no correlation among the observations between the blocks and correlation structure between plots within a circular block to be the same in each block and the structure of W_k will be,

$$W_k = \begin{bmatrix} 1 & \rho & 0 & \cdots & \rho \\ \rho & 1 & \rho & 0 & 0 \\ \vdots & \rho & \ddots & \rho & \vdots \\ 0 & \cdots & \rho & 1 & \rho \\ \rho & 0 & \cdots & \rho & 1 \end{bmatrix}$$

(1.2)

where $\rho(-1 \le \rho \le +1)$ is the correlation coefficient between the neighbouring plots in a block. Let us consider the above design set-up in the line of Gill and Shukla (1985). Then, the information matrix (C matrix) for estimating the treatment effects having correlated observations estimated by generalized least squares will be as follows:

$$C = \left[X'V^{-1}X\right] - \left[X'V^{-1}Z\left(Z'V^{-1}Z\right)^{-1}Z'V^{-1}X\right] \quad \textbf{(1.3)}$$

The above C matrix (1.3) for estimating the effect of treatments in a block design is symmetric, non-negative definite with zero row and column sums.

Let us consider only 2 associate class Generalised Neighbour Partially Balanced Incomplete Block (GNPBIB2) designs with parameters v, b, r, k, n_1, n_2, λ_{11}, λ_{12}, λ_{21} and λ_{22} for the results in the next section.

2. Important results on GNPBIB2 designs for correlated observations

According to model (1.1) with correlated errors as given in 1.2, the matrix $X'V^{-1}Z$ will be considered as the treatment vs. block incidence matrix of order v×b. Here it should be noted that the structure of the matrix $[(X'V^{-1}Z) \times (Z'V^{-1}X)]$ (or $N^*N^{*\prime}$) is identical to structure of the NN' of any two associate class PBIB designs, where N is the treatment vs. block incidence matrix of order v × b of the PBIB design. Thus, the diagonal elements of the matrix $N^*N^{*\prime}$ are all equal to $r(1+2\rho)^2$ and in each row of $N^*N^{*\prime}$ there are precisely n_1 positions filled by $\lambda_{11}(1+2\rho)^2$ and precisely n_2 positions filled by $\lambda_{12}(1+2\rho)^2$. Again, the diagonal elements of the matrix $X'V^{-1}X$ (or $r^{*\delta}$) will be r and in each row of $r^{*\delta}$ there are precisely n_1 positions filled by $\rho\lambda_{21}$ and precisely n_2 positions filled by $\rho\lambda_{22}$. In the design, the matrix $Z'V^{-1}Z$ (or $k^{*\delta}$) = $[k(1+2\rho) \times \mathbf{I}]$, where **I** is an identity matrix of order b.

Lemma 2.1: The diagonal elements of the C-matrix (1.3) of a GNPBIB2 design will be

$$C_{ii} = \frac{r[k^2 - k(1+2\rho)]}{k^2}$$ and in each row of C, there are precisely n_1 positions filled by

$$C_{ij_1} = \frac{-\lambda_{11}k(1+2\rho) + k^2\rho\lambda_{21}}{k^2},$$

(j_1 =1, 2,…, n_1) and precisely n_2 positions filled by

$$C_{ij_2} = \frac{-\lambda_{12}k(1+2\rho) + k^2\rho\lambda_{22}}{k^2}, \quad (j_2 = 1, 2, \ldots, n_2).$$

Proof.: According to the model with correlated observations (Gill and Shukla, 1985) the structure of C matrix has been given in 1.3. The proof is straight forward for the GNPBIB2 design with parameters v, b, r, k, n_1, n_2, λ_{11}, λ_{12}, λ_{21} and λ_{22}.

Hereinafter, considered only the two class triangular GNPBIB2 designs called as $TGNPBIB_2$ designs.

2.1. Triangular two-class association scheme:

Let there be v = n(n-1)/2 treatments, arranged in square array of side n, such that the position on principle diagonal of the array are left blank, the n(n-1)/2 positions above the principle diagonal are filled up by v treatment symbols in such a manner that the resultant arrangement is symmetrical about principal diagonal. The two-class triangular (T_2) association scheme then has the following association rule: two treatments are first associates if they belonging to the same row or same column of the array and are second associates otherwise.

The parameters of the T_2 association scheme are as follows

$v = n(n-1)/2$, $n_1 = 2(n-2,)$ $n_2 = (n-2)(n-3)/2$,

$$P_1 = \begin{bmatrix} \text{n-2} & \text{n-3} \\ \text{n-3} & \text{(n-3)(n-4)/2} \end{bmatrix}$$

$$P_2 = \begin{bmatrix} 4 & \text{2n-8} \\ \text{2n-8} & \text{(n-4)(n-5)/2} \end{bmatrix}$$

where n $\leq$ 5. For n = 4, the triangular scheme reduced to Group Divisible (GD) scheme.

2.2. Triangular GNPBIB$_2$ (TGNPBIB$_2$) designs for correlated observations:

A GNPBIB2 design is said to be Triangular design (or TGNPBIB$_2$), if it is based on T_2 association scheme. Let $N^* = (X'V^{-1}Z)$ be the incidence matrix of the TGNPBIB$_2$ with parameters $v = n(n-1)/2$, b, r, k, $n_1 = 2(n-2)$, $n_2 = (n-2)(n-3)/2$, λ_{11}, λ_{12}, λ_{21} and λ_{22}.

Lemma 2.2.1: The row or column sum of the matrix $[(X'V^{-1}Z) \times (Z'V^{-1}X)]$ (or $N^*N^{*\prime}$) of the above TGNPBIB$_2$ design will be $rk(1+2\rho)^2$ and determinant of the matrix $N^*N^{*\prime}$ will be

$$rk\left(1+2\rho\right)^2\left[\left(r-2\lambda_{11}+\lambda_2\right)(1+2\rho)^2\right]^{n(n-3)/2}$$

$$\left[(r+(n-4)\lambda_{11}-(n-3)\lambda_{12})(1+2\rho)^2\right]^{(n-1)}.$$

Proof: The result is straight forward following the steps given in Raghavarao (1960) on the matrices developed by model 1. 1 and 1.2.

The eigen roots (θ_{1i}, i= 0, 1, 2) with the respective multiplicity α_i, (i= 0, 1, 2) of the matrix $N^*N^{*\prime}$ (Lemma 2.2.1) will be

$$\theta_{10} = rk(1+2\rho)^2, \qquad \alpha_0 = 1; \qquad \textbf{(2.2.1)}$$

$$\theta_{11} = \left(r+(n-4)\lambda_{11}-(n-3)\lambda_{12}\right)(1+2\rho)^2, \quad \alpha_1 = n-1; \qquad \textbf{(2.2.2)}$$

$$\theta_{12} = \left(r-2\lambda_{11}+\lambda_{12}\right)(1+2\rho)^2, \quad \alpha_2 = n(n-3)/2; \qquad \textbf{(2.2.3)}$$

Similarly for $(X'V^{-1}X)$ (or r^{*a}) matrix of TGNPBIB$_2$ designs, the eigen roots (values) (θ_{2i}, i= 0, 1, 2) with respective multiplicity α_i (i= 0,1,2) will be

$$\theta_{20} = r + (n_1\lambda_{21} + n_2\lambda_{22})\rho = r(1+2\rho), \quad \alpha_0 = 1; \qquad \textbf{(2.2.4)}$$

$$\theta_{21} = r + \left[(n-4)\lambda_{21}-(n-3)\lambda_{22}\right]\rho, \quad \alpha_1 = n-1; \qquad \textbf{(2.2.5)}$$

$$\theta_{22} = r - \left[2\lambda_{21}+\lambda_{22}\right]\rho, \quad \alpha_2 = n(n-3)/2; \qquad \textbf{(2.2.6)}$$

Considering the results of 2.2 section, the following lemma can be observed.

Lemma 2.2.2: The eigen roots (values) of information matrix (C) of TGNPBIB$_2$ design with parameters $v = n(n-1)/2$, b, r, k, $n_1 = 2(n-2)$, $n_2 = (n-2)(n-3)/2$, λ_{11}, λ_{12}, λ_{21} and λ_{22} will be

$$\kappa_0 = \left[\theta_{20} - \frac{\theta_{10}}{k(1+2\rho)}\right] = 0 \text{ with multiplicity 1;}$$

$$\kappa_1 = \left[\left(\frac{r(k-1)+\lambda_{11}}{k}\right) - \left(\frac{\{2r+\lambda_{12}k-2\lambda_{11}\}\rho}{k}\right)\right]$$

with multiplicity α_1 and

$$\kappa_2 = \left[\left(\frac{v\lambda_{12}}{k}\right) - \left(\frac{\{2\lambda_{12}-k\lambda_{22}\}\rho}{k}\right)\right]$$

with multiplicity α_2.

Proof: Results are straight forward following the results from 2.2.1 to 2.2.6.

3. Method of construction for TGNPBIB$_2$ Designs:

An extensive list of PBIB designs with two associate classes was prepared by Clatworthy (1973). The listed designs or repetition of the designs of Clatworthy (1973) are used to construct TGNPBIB$_2$ designs in circular blocks with little adjustment of positions of treatments in a block or with no adjustment.

Method 3.1: A series of TGNPBIB$_2$ designs in circular blocks having parameters [$v = n(n-1)/2$, b, r, k ($\geq$ 4), $n_1 = 2(n-2)$, $n_2 = (n-2)(n-3)/2$, λ_{11}, λ_{12}, λ_{21} and λ_{22}] be constructed from the T_2 designs or the repetition of the T_2 designs of Clatworthy (1973) with parameters [$v = n(n-1)/2$, b, r, k ($\geq$ 4), $v = n(n-1)/2$, λ_1 λ_2] after a little adjustment of positions of treatments in a block or without adjustment, if the following conditions are satisfied.

1. For any two integers **s** and **t** (both should not be zero simultaneously), then $(sn_1+tn_2) \mid 2r$.

2. $\sum_i n_i \lambda_{2i} = 2r$, i= 1, 2.

Proof: Let the 1st (or 2nd) associate treatments of a particular treatment be appear as neighbour in the design **s** (or **t**) number of times, where **s** and **t** be either zero or any positive integer without assuming zero simultaneously. The total number of neighbours in a TGNPBIB$_2$ design with respect to any treatment will be 2r. This 2r should be divisible by the occurrence of 1st and 2nd associate treatments as neighbours. Thus the condition 1 is satisfied. The proof of condition 2 is already given in definition 1.4.

Example 3.1: Consider a little adjustment of positions of treatments in a Triagular PBIB$_2$ design (T28, Clatworthy, 1973) with parameters v = 10, b = 5, r = 2, k = 4, n_1=6, n_2=3, λ_1=1 & λ_2=0. Then PBIB design becomes a TGNPBIB$_2$ design in circular blocks having parameters (v = 10, b = 15, r = 6, k = 4, n_1=6, n_2=3, λ_{11}=3, λ_{12}=0, λ_{21}=2 and λ_{22}=0), Here, 2r=12 and $2r/(sn_1+tn_2)$= 12/(6+0)=2; where s=1 & t=0, whose solution is given below:

4	1	2	3	4	1		4	2	1	3	4	2		4	2	3	1	4	2
1	5	6	7	1	5		1	6	5	7	1	6		1	6	7	5	1	6
5	8	9	2	5	8		5	9	8	2	5	9		5	9	2	8	5	9
8	10	3	6	8	10		8	3	10	6	8	3		8	3	6	10	8	3
10	4	7	9	10	4		10	7	4	9	10	7		10	7	9	4	10	7

Example 3.2: Consider a little adjustment of positions of treatments in a Triagular PBIB$_2$ design (T48, Clatworthy, 1973) with parameters v = 15, b = 6, r = 2, k = 5, n_1=8, n_2=6, λ_1=1 & λ_2=0. Then PBIB design becomes a TGNPBIB$_2$ design in circular blocks having parameters (v = 15, b = 12, r = 4, k = 5, n_1=8, n_2=6, λ_{11}=2, λ_{12}=0, λ_{21}=1 and λ_{22}=0), Here, 2r=8 and $2r/(sn_1+tn_2)$= 8/(8+0)=1; where s=1 & t=0, whose solution is given below:

5	1	2	3	4	5	1		5	2	4	1	3	5	2
1	6	7	8	9	1	6		1	7	9	6	8	1	7
6	10	11	12	2	6	10		6	11	2	10	12	6	11
10	13	14	3	7	10	13		10	14	7	13	3	10	14
13	15	4	8	11	13	15		13	4	11	15	8	13	4
15	5	9	12	14	15	5		15	9	14	5	12	15	9

4. Efficiency of TGNPBIB$_2$ designs for correlated observations

For a connected block design d, let κ_1, κ_2,…, κ_{v-1} , be the non-zero (v-1) eigen values of C-matrix of the design. The design is universally optimal as defined by Kifer (1975), if all κ_i's are equal with maximum trace of C-matrix. Thus, the above TGNPBIB$_2$ designs are not universally optimum like any TPBIB$_2$ designs.

Now define $\varphi_A(d)=\frac{1}{v-1}\sum_{i=1}^{v-1}\kappa_i^{-1}$ and $\varphi_D(d)=\left(\prod_{i=1}^{v-1}\kappa_i^{-1}\right)^{\frac{1}{v-1}}$.

Then, a design is A- [D-] optimal, if it maximizes the φ_A(d) [φ_D(d)] over the class D (v, b, k). The A and D efficiencies of a design d over the class D (v, b, k) given by Gill and Shukla (1985), as:

$$e_A(d) = \frac{\varphi_A(d^*)}{\varphi_A(d)}$$

$$e_D(d) = \frac{\varphi_D(d^*)}{\varphi_D(d)}$$

Where d^* is a hypothetical universally optimal design whose information matrix C has equal nonzero eigen values *i.e.*, $\frac{1}{v-1}\sum_{i=1}^{v-1}\kappa_i^{-1}$ with multiplicity v-1.

Result 4.1: If $\frac{\kappa_1}{\kappa_2} = \Psi$, be a constant for different values of ρ ($-1 \le \rho \le +1$) of a TGNPBIB$_2$ design, then the efficiencies (A and D) of the design will remain constant over different values of ρ .

Table 1. A- and D- efficiency of T$_2$ Generalized Neighbour Partially Balanced Incomplete block design of series of method 3.1 for correlated observations.

v	b	r	k	λ_{11}	λ_{12}	n	n_1	n_2	λ_{21}	λ_{22}	ρ	A-efficiency	D-efficiency
10	15	6	4	3	0	5	6	3	2	0	0	0.947	0.947
15	12	4	5	2	0	6	8	6	1	0	0	0.942	0.942

Note: Efficiencies (A & D) are independent to ρ values ($+1 \le \rho \le -1$), see also Result 4.1.

Proof: Let $TGNPBIB_2$ be a design with parameters $v = mn, b, r, k, n_1 = n-1, n_2 = n(m-1)$, λ_{11}, λ_{12}, λ_{21} and λ_{22}; its C matrix has $\alpha_1 + \alpha_2$ nonzero eigen values. Let eigen values be denoted by κ_1 and κ_2 with multiplicity α_1 and α_2, respectively. Let us consider the above definitions of optimality, φ_A and φ_D values for d and d*.

If $\frac{\kappa_1}{\kappa_2} = \psi$, => $\kappa_1 = \psi\kappa_2$, then the efficiencies (A and D) of a design d over d* will be

$$e_A(d)=\left[\frac{(v-1)^2\psi}{\psi\{2m(m-1)-v(2m-v)+1\}+\{(1+\psi^2)(m^2-m)(n-1)\}}\right]$$

and $$e_D(d)=\left[\frac{(v-1)\left(\psi^{\frac{m-1}{v-1}}\right)}{\{\psi(m-1)+m(n-1)\}}\right]$$

When ψ is a constant for every value of ρ, then these efficiency values remain same over the different ρ values because efficiencies are the function of ψ value and not the function of ρ. Actually, it is also observed that if in a $TGNPBIB_2$ design, the following relation holds, then the efficiencies (A and D) of the design will remain constant over different values of ρ.

$$\left[2(n_2\lambda_{12}^2+n_1\lambda_{11}\lambda_{12})-k\lambda_{11}\lambda_{22}(n_1+1)-n_2\lambda_{12}\lambda_{22}(k-1)+\lambda_{12}\lambda_{21}(n_1+k)\right]=0$$

Remark 1: Two associate class PBIB designs with any association scheme for k = 3, will be a GNPBIB2 as in definition 1.4.

The Efficiency values (A and D) of example designs (3.1 & 3.2) are presented in table 1 for different values of ρ ($-1 \leq \rho \leq +1$).

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